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Duality, phases, spinors and monopoles in SO(N) and Spin(N) gauge theories

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Abstract: Four-dimensional N=1 supersymmetric $Spin(N)$ gauge theories with matter in the vector and spinor representations are considered. Dual descriptions are known for some of these theories. It is noted that when masses are given to all fields in the spinor representation, the dual gauge group G breaks to a group H such that $\pi_2(G/H) = Z_2$. The quantum numbers of the associated Z_2 monopole and those of the massive spinors are shown to agree, suggesting that the monopole is the image of the massive spinors under duality. It follows that electric sources in the spinor representation, needed as test charges to determine the phase of an $SO(N)$ gauge theory, can be introduced as Z_2-valued magnetic sources in the dual nonabelian gauge theory. This fact is used to study the phases of $SO(N)$ gauge theories with matter in the vector representation.

Keywords: Solitons Monopoles and Instantons, Nonperturbative Effects, Confinement, Supersymmetry and Duality.

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Contents

1. Overview

Duality in field theory and string theory has become a central area of research in recent years. Although a great deal has been learned at both the technical and the conceptual level, there are still many remaining questions. In this paper I examine some outstanding issues involving the physics of $\mathcal{N} = 1$ supersymmetric gauge theories in four dimensions.

After the original Seiberg-Witten solutions to $\mathcal{N} = 2$ gauge theory were found [1, 2], duality was discovered in $\mathcal{N} = 1$ nonabelian supersymmetric gauge theories by

Seiberg [3]. In [3] and [4], the physics of duality for $SO(N)$ gauge theories[1] with N_f fields in the vector representation was studied. The $SO(N)$ gauge theories were shown to have magnetic and sometimes dyonic dual descriptions. The different phases of these theories were described. For $N_f \geq 3(N-2)$ the theories are not asymptotically free and are described as being in the "free electric phase". In this case their dual descriptions are strongly coupled. For $3(N-2) > N_f > \frac{3}{2}(N-2)$, the theory moves into the "non-abelian Coulomb phase" where all descriptions are strongly coupled and the low energy physics is that of a non-trivial conformal field theory. At still lower N_f the theory becomes very strongly coupled, while one of its dual descriptions loses asymptotic freedom and becomes a good weakly coupled description in the infrared. This description has an $SO(N_f - N + 4)$ gauge group. If the dual gauge group is unbroken, this phase is called the "free magnetic phase" since magnetic degrees of freedom are free at long distances. If the dual description is an abelian gauge theory ($N_f = N-2$) then the original description is in the "Coulomb phase", a special case of the free magnetic phase. For $N_f = N-3$ the theory enters the "confining phase". The weakly coupled dual gauge theory is completely broken by the Higgs mechanism; from the point of view of the original description, the fields which have condensed are magnetically charged, leading to confinement of the original fields. Similar behavior persists for $N_f = N-4$. For lower values of N_f (except $N_f = 0$) the theory has no vacuum.

The arguments of [3, 4] were based on a interwoven assemblage of powerful circumstantial evidence. It would be nice, however, to strengthen the arguments further, particularly in the discussion of the various phase structures and their properties. To this end, it would be especially useful to be able to introduce sources which are in the spinor representation of the gauge group. Such sources have charges which cannot be screened by massless fields of the $SO(N)$ theory, and so a Wilson loop in this representation should be a good diagnostic for the phase of the theory. How can the Wilson loop in the spinor representation be introduced into the dual description of the theory? How is the electrically charged source mapped under duality? If we study $SO(2)$ without matter, then the answer is known; as a consequence of the usual electric-magnetic duality of the free classical Maxwell equations, the electrically charged source of the first description is simply a magnetically charged source of the other. But such a straightforward transformation is not possible in the non-abelian case. Certainly the appropriate duality transformation cannot be visible in the classical equations of the theory.

To resolve this question, I continue with my historical review. Pouliot soon discovered that $Spin(7)$ with fields in the spinor representation is dual to a chiral, $SU(N)$ gauge theory [5]. Further generalizations of this theory then followed. In [6], Pouliot

[1] The distinction between $SO(N)$ and $Spin(N)$, its double cover, is essential for the topological arguments used in this paper.

and the author showed that $Spin(8)$ with N_f fields in the $\mathbf{8_v}$ and one field in the $\mathbf{8_s}$ representation is dual to a chiral $SU(N_f-4)$ gauge theory.[2] Furthermore, when the $\mathbf{8_s}$ is given a mass, the dual $SU(N_f-4)$ theory is broken to $SO(N_f-4)$, which is a dual description of $Spin(8)$ with N_f fields in the $\mathbf{8_v}$.

In this we see the answer to the question posed above. When $SU(N_f-4)$ breaks to $SO(N_f-4)$, a topologically stable monopole carrying a Z_2 charge is found in the theory [7]. This is because $\pi_2[SU(N)/SO(N)] = \pi_1[SO(N)] = Z_2$, for $N > 2$. As I will argue below, the massive spinor of the $Spin(8)$ theory is mapped under duality to the Z_2 monopole of the broken $SU(N_f-4)$ theory. Consequently, the Wilson loop in the spinor representation of $Spin(8)$ is mapped to the 't Hooft loop in the magnetic Z_2 representation of the low-energy $SO(N_f-4)$ gauge theory. This is natural, since all charges of the spinor, except for its Z_2 charge under the $Z_2 \times Z_2$ center of $Spin(8)$, will be screened by the light fields in the vector and adjoint representations.

The work of [6] was further generalized to $Spin(10)$ gauge theories with a number of fields in the **10** representation and one [8] or more [9] fields in the **16** representation. Although masses for **16**'s cannot be introduced in $Spin(10)$, they can be added when the theory is broken to a smaller $Spin$ group. The dual theories always contain an SU factor, which is broken to an SO gauge theory if and only if all spinors are massive. Again the spinors appear as monopoles carrying a Z_2 charge. More details will be given in sections 2 and 3.[3]

An aside about notation: since the magnetic monopoles in question are found in theories which are themselves "magnetic" descriptions of theories that may be in the free magnetic phase, there is obvious room for confusion. I therefore will abandon the commonly used terminology of "electric theory" and "magnetic theory", which are conventional in any case. Instead I will refer to the $Spin(N)$ model with the massive field(s) in the spinor representation as the "A theory", and to its dual as the "B theory".

Returning to the physics, one can use the insight described above to study the phases and dualities of these models. For example, if the $Spin(8)$ theory with a massive spinor is in the free magnetic phase, so that the low-energy $SO(N_f-4)$ gauge group of the B theory is weakly coupled in the infrared, then semiclassical physics of the B theory monopoles implies that the spinors of the A theory are unconfined and have a $(\log r)/r$ potential between them. If some additional fields in the A theory are given masses, so that the A theory enters the confining phase, then the B theory gauge group is completely broken via the Higgs mechanism, a Nielsen-Olesen string soliton [11] is present, and the spinors/monopoles are explicitly confined by a linear potential. A more detailed discussion of phases, supporting previous results of [3, 4, 15], will be given in section 4.

[2]This and the other duality transformations used in this paper are summarized in the Appendix.

[3]Although these theories certainly have states with both electric and magnetic charge, I have chosen not to discuss dyons and dyonic duals[4, 15] in this paper.

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It is amusing that the $SU \to SO$ breaking pattern and the topological relation $\pi_2[SU(n)/SO(n)] = Z_2$ have appeared before in the context of strongly coupled $SO(N)$ gauge theories, specifically in Witten's work on current algebra [12]. It is perhaps important to emphasize the differences, to avoid confusion. In Witten's conjecture, the $SU(n)$ and $SO(n)$ groups in the coset are flavor symmetries of matter fields, with $n = N_f$, and with the coset being the pion moduli space resulting from chiral symmetry breaking. This breaking leads to a finite tension global string soliton – a two-dimensional Skyrmion – which Witten suggested was the string responsible for confinement of spinors. Here the situation is very different. The $SU(n)$ and $SO(n)$ groups are gauge groups of a dual description; all physical states are invariant under them. The coset is due not to chiral symmetry breaking, which does not occur in these gauge theories, but to spontaneous gauge symmetry breaking driven by a parameter in the Lagrangian or an expectation value for a gauge singlet field. The topology of gauge breaking leads to an *unconfined* monopole in the B theory. Confinement only occurs if the $SO(n)$ group is itself completely broken. The string which is responsible for confinement of spinor particles is a Nielsen-Olesen string of the dual theory, not a Skyrmion string built on the space of gauge invariant vacua.

2. General expectations

Let us first consider what we might expect the physics to look like. If the A theory is strongly coupled, the fields of the A theory should not, in general, be visible in the B theory. The quarks of QCD cannot be seen in the physics of the chiral Lagrangian; electrons cannot be seen in the Landau-Ginsburg theory of superconductivity. Massless or light states should be especially difficult to find, as they are deeply involved in the dynamics of the theory. Even massive particles may not be visible if all of their charges can be screened. For example, if both massive and massless spinor representations of $Spin(N)$ are present, a massive spinor will generate a massless spinor cloud around it, even if the dynamics are relatively weakly coupled. These neutral bound states of the A theory will be visible as gauge singlets of the B theory, but the original spinor fields themselves will not be visible.

By contrast, if the A theory has massive spinors but no massless ones, then the fact that spinor representations have a charge under the center of $Spin(N)$ becomes important. The light fields of the theory are all representations of $SO(N)$ and so are neutral under the group $Z_2 = Spin(N)/SO(N)$. They therefore cannot screen the Z_2 charge of the massive spinors. Although a given spinor can surround itself with a cloud of light fields, its Z_2 charge cannot be removed. On the other hand, a state containing two spinors need not be visible (if the spinors are not widely separated) since the two as a pair may be screened by the light fields.

One may then expect that the lightest state with non-zero Z_2 charge might be visible in the B theory. Since this state must disappear both when the spinor mass is taken to infinity and when it is taken to zero, it is natural to expect the masses of the B theory state and that of the A theory spinor are correlated. But there is no reason to expect the masses to be related in a simple way; the binding energy between the spinor and its surrounding cloud may be very large.

What if the A theory has many spinors? If all of the spinors have the same mass, then the lightest states in the A theory with non-zero Z_2 charge have a certain degeneracy. It is natural to expect this degeneracy to be visible in the B theory as well; the flavor symmetries of the lightest spinors ought to be visible in the B theory.

In conclusion, the B theory may be expected to contain heavy states with a Z_2 charge, whose mass is correlated with the spinor mass(es), and whose flavor symmetries match those of the spinors.

It is natural that the state carrying this Z_2 charge be a magnetic monopole, given the situation in $\mathcal{N}=2$ gauge theories [2]. Consider $SO(3)$ with a massless triplet and $2N_f < 8$ doublets of mass m, coupled to the triplet in an $\mathcal{N}=2$-symmetric fashion, as the A theory. Quantum mechanically the $SO(3)$ gauge group is broken to $SO(2)$, under which the massive doublets have charge $\pm 1/2$. A Maxwell electric-magnetic duality transformation on the $SO(2)$ converts the description to that of the B theory. The electrically charged massive doublets become heavy Dirac magnetic monopoles, with magnetic charge $\pm 1/2$, of the B theory. Since the light fields of the A theory, the triplet and the gauge bosons, have charge $0, \pm 1$, the heavy doublets/monopoles carry a conserved Z_2 quantum number. The situation considered in this paragraph will emerge as a special case of the results given below.

3. Flavor representations

The strongest evidence that the massive particle in the spinor representation of the A theory appears in the B theory as a monopole comes from the transformation properties of the spinor and monopole under flavor groups of the two theories. It will also become clear in this section that monopoles only arise in the B theory when *all* of the spinors in the A theory are massive, consistent with the discussion of section 2.

3.1. The massive spinor of $Spin(8)$

Consider, as the A theory, $Spin(8)$ with N_f fields V^i in the $\mathbf{8_v}$ and a single massive spinor P in the $\mathbf{8_s}$. The superpotential is $W = 1/2mPP$. For bookkeeping purposes, let us separate the V fields into V^i, $i = 1, \dots, N_f - k$, and $\hat{V}^r$, $r = 1, \dots, k$. We may then go to the point in moduli space where $\langle \hat{V}^r \hat{V}^s \rangle = v^2 \delta^{rs}$. In this vacuum the gauge symmetry is broken to $Spin(8-k)$ and the $SU(N_f)$ flavor symmetry is broken to $SU(N_f - k) \times SO(k)$. Actually the last factor, which is a diagonal subgroup of the original flavor and color groups, is $Spin(k)$; the field P becomes a (generally reducible) eight-dimensional bispinor under $Spin(8-k) \times Spin(k)$. For example, if $k=2$, then P is a $(\mathbf{4}, +) + (\bar{4}, -)$ of $Spin(6) \times Spin(2)$, while if $k=3$, then P is a $(\mathbf{4}, \mathbf{2})$ of $Spin(5) \times Spin(3)$.

As described in the Appendix, the B theory has a $SU(N_f-4)$ gauge group, with matter S in the symmetric representation and $Q_i, \hat{Q}_r$ in the antifundamental representation. It also has gauge singlets N^{ij}, N^{ir}, N^{rs} and T. When the singlet $N^{rs} = V^r V^s$ has an expectation value $N^{rs} = v^2 \delta^{rs}$, the effective superpotential becomes

$$W = \sum_{i,j=1}^{N_f-k} \frac{1}{\mu_1^2} N^{ij} Q_i S Q_j + \sum_{r=1}^{k} \frac{v^2}{\mu_1^2} \hat{Q}_r S \hat{Q}_r + \frac{1}{\mu_2^{N_f-5}} T \det S + mT \,. \tag{3.1}$$

The μ_i are parameters of dimension one needed for dimensional consistency (and other issues) [4, 15]; their presence and physical meaning will be irrelevant for this paper. The singlets N^{ir} are the images under duality of the $Spin(8-k)$-singlet components of the V^i; they have decoupled from this superpotential in the standard way with the help of a field redefinition. The flavor group is clearly broken to $SU(N_f-k) \times SO(k)$, in agreement with the A model. The F-flatness condition $\partial W / \partial T = 0$ ensures that $\langle \det S \rangle$ is nonzero, and so $SU(N_f-4)$ is broken to $SO(N_f-4)$, leading to a Z_2 monopole [7]. As a consequence of their coupling to S, the $\hat{Q}_r$ become massive, and they each have a single zero mode in the presence of the monopole. These zero modes transform in the vector representation of $SO(k)$, and therefore, after quantization, the Z_2 monopole will transform as a spinor of $Spin(k)$. This agrees with the global charges of P.

3.2. The massive spinor of $Spin(10)$

Next consider, as the A model, $Spin(10)$ with N_f fields V^i in the $\mathbf{10}$ and a single spinor P in the $\mathbf{16}$. Since the spinor representation is chiral, we cannot write a mass term for P, but mass terms can be written if $Spin(10)$ is broken to a smaller $Spin$ group; in $Spin(10)$ language, P may become massive by coupling to a vector which acquires an expectation value. Again let us separate the fields into V^i, $i = 1, \ldots, N_f - k$, and $\hat{V}^r$, $r = 1, \ldots, k$. We will take the superpotential to be $W = y \hat{V}^1 PP$. At the point in moduli space where $\langle \hat{V}^r \hat{V}^s \rangle = v^2 \delta^{rs}$, the spinor P has mass yv, the gauge symmetry is broken to $Spin(10-k)$ and the $SU(N_f)$ flavor symmetry is broken to $SU(N_f-k) \times Spin(k-1)$. For very small y the last factor is $Spin(k)$, and we may take the limit of large v and small y, holding yv fixed, for the purposes of discussing quantum numbers. The field P decomposes into a generally reducible sixteen-component bispinor representation of $Spin(10-k) \times Spin(k)$. For example, if $k=2$, then p is a $(\mathbf{8_s}, +) + (\mathbf{8_c}, -)$ of $Spin(8) \times Spin(2)$, while if $k=5$, then p is a $(\mathbf{4}, \mathbf{4})$ of $Spin(5) \times Spin(5)$.

As reviewed in the Appendix, the B theory has $SU(N_f-5)$ gauge group with matter S in the symmetric tensor representation, $Q_i, \hat{Q}_r$ in the antifundamental representation, and F in the fundamental representation, along with gauge singlets N^{ij}, N^{ir}, N^{rs} and

$Y^i, \hat{Y}^r$. When the singlet N^{rs} has an expectation value $N^{rs} = v^2\delta^{rs}$, the effective superpotential becomes

$$W = \frac{\det S}{\mu_2^{N_f-8}} + \sum_{i,j=1}^{N_f-k} \frac{1}{\mu_1^2} N^{ij} Q_i S Q_j + \sum_{r=1}^{k} \frac{v^2}{\mu_1^2} \hat{Q}_r S \hat{Q}_r + \\ + \frac{1}{\mu_3^2}(\sum_{i=1}^{N_f-k} Y^i Q_i + \sum_{r=1}^{k} \hat{Y}^r \hat{Q}_r) F + y\hat{Y}^1 . \qquad (3.2)$$

The flavor group is broken to $SU(N_f - k) \times SO(k-1)$, with the latter factor extended to $SO(k)$ for small y, in agreement with the A model. The F-flatnesss condition $\partial W/\partial Y^1 = 0$ equation for Y^1 ensures that $\langle \hat{Q}_1 F\rangle$ is non-zero, breaking $SU(N_f - 5)$ to $SU(N_f - 6)$. Under this breaking S decomposes into a symmetric tensor s, a fundamental f, and a singlet z. Through the condition $\partial W/\partial z = 0$ he expectation value for $\hat{Q}_1$ then requires $\det s \propto \hat{Q}_1\hat{Q}_1 \propto y\mu_3^2$. This breaks the theory to $SO(N_f - 6)$, leading to a Z_2 monopole [7]. As a consequence of their coupling to S, the $\hat{Q}_r$ become massive when $\det S$ gets an expectation value, and they each have a single zero mode in the presence of the monopole. Since these zero modes transform in the vector representation of $SO(k)$, the Z_2 monopole will transform as a spinor of $Spin(k)$. As before, this agrees with the global charges of P.

As an additional check, let us examine what happens if we take $k = 2$, and let the fields $\hat{V}^r$ have expectation values such that $\langle V^1V^1\rangle = \langle V^2V^2\rangle = 0$, $\langle V^1V^2\rangle = v^2$. The A theory is broken to $Spin(8)$, with $N_f - 2$ vectors V^i in the $\mathbf{8_v}$, and with the spinor P decomposing into a field P_s in the $\mathbf{8_s}$ and a field P_c in the $\mathbf{8_c}$. The $Spin(10)$ superpotential we take to be

$$W = [y_s\hat{V}^1 + y_c\hat{V}^2]PP . \qquad (3.3)$$

One can check that in the low-energy $Spin(8)$ theory, the mass of P_s is proportional to $y_s v$ and that of P_c is proportional to $y_c v$. The flavor symmetry of the model is $SU(N_f - 2) \times Spin(2)$ if $y_s = y_c$, with the $Spin(2)$ broken otherwise; P_s and P_c have opposite charge under the $Spin(2)$. It is interesting to consider taking one mass to zero holding the other fixed, or alternatively holding the first fixed and taking the other to infinity. What happens, in these two limits, in the B theory?

The B theory has superpotential

$$W = \frac{\det S}{\mu_2^{N_f-8}} + \sum_{i,j=1}^{N_f-2} \frac{1}{\mu_1^2} N^{ij} Q_i S Q_j + \frac{v^2}{\mu_1^2} \hat{Q}_1 S \hat{Q}_2 + \\ + \frac{1}{\mu_3^2}(\sum_{i,j=1}^{N_f-2} Y^i Q_i + \hat{Y}^1\hat{Q}_1 + \hat{Y}^2\hat{Q}_2) F + y_s\hat{Y}^1 + y_c\hat{Y}^2 . \qquad (3.4)$$

The F-term equations for $\hat{Y}^1, \hat{Y}^2$ imply that $\hat{Q}_1, \hat{Q}_2, F$ get expectation values, which by the D-term equations must all lie along the same direction in the color group, breaking the gauge group to $SU(N_f-6)$. The field S decomposes as above, and the condition $\partial W/\partial z=0$ then requires $\det s \propto Q_1Q_2 \propto y_s y_c \mu_3^2/\sqrt{|y_s|^2+|y_c|^2}$. This breaks the gauge group to $SO(N_f-6)$ and generates a Z_2 monopole solution. The flavor group includes $SU(N_f-2)$, with an additional $SO(2)$ factor if $y_s=y_c$. Thus, only when $y_s=y_c$ must the stable monopole be a doublet of $SO(2)$, according with the A theory. Furthermore, if either one is massless, it can screen the other, making both invisible. We can see these effects in the limits $y_s y_c = 0$ or ∞. As we take y_s or y_c to zero, holding the other fixed, so that the $SU(N_f-6)\to SO(N_f-6)$ breaking scale becomes very low, the monopole becomes light, disappearing in the $y_s y_c=0$ limit where the $SU(N_f-6)$ symmetry is restored. In the limit y_s or y_c goes to infinity, with the other held fixed, the $SU(N_f-6)\to SO(N_f-6)$ breaking scale goes to a constant, keeping the monopole at a finite mass.

One may perform a similar analysis by breaking the $Spin(10)$ theory with one spinor to $Spin(4)$, leaving a $Spin(6)$ flavor group. The spinors transform in the $(\mathbf{2},\mathbf{4})$ and $(\bar{\mathbf{2}},\bar{\mathbf{4}})$ representations of the $Spin(4)\times Spin(6)$ group. If half the spinors are much more massive than the others, the flavor group is reduced to $Spin(5)$, with the light spinors transforming in the $(\mathbf{2},\mathbf{4})$ of the $Spin(4)\times Spin(5)$ group. It can be easily checked that in the process the $SO(6)$ flavor group of the B theory is reduced to $SO(5)$, leaving the monopole with the correct quantum numbers.

3.3. Multiple spinors

Finally, consider $Spin(10)$ with N_P fields P_a in the $\mathbf{16}$ representation and N_f fields V^i in the $\mathbf{10}$ representation [9]. After breaking $Spin(10)$ to $Spin(9)$ or below, one may add masses for some or all of the spinor fields and study their properties in the B theory. However, because the B theory's response to these perturbations is extremely complicated, I will not present the details of the analysis, giving instead only a sketch of the procedure. First break the A theory to $Spin(9)$; this does not change the gauge group of theory B, although it does change its superpotential. Each spinor of $SO(10)$ becomes a spinor of $SO(9)$. Giving masses to all but n of these spinors leads the B theory gauge group to break from $SU(N_f+2N_P-7)\times Sp(2N_P-2)$ to $SU(N_f+2n-7)\times Sp(2n-2)$. If $n=1$ we obtain the $SU(N_f-5)$ dual theory discussed earlier, while if $n=0$ the B theory gauge group breaks further to $SO(N_f-6)$. (The reason we do not find $SO(N_f-5)$ is associated with the alteration to the B superpotential which accompanies the breaking of $SO(10)$ to $SO(9)$ [8, 9]; note this is consistent with Seiberg duality [3, 4].) The topological properties of the breaking patterns do not predict a Z_2 monopole except in the case $n=0$, for which $\pi_2[SU(N_f+2n-7)\times Sp(2n-2)/SO(N_f-6)]=\pi_1[SO(N_f-6)]=Z_2$. If any of the spinors are massless, the breaking to an SO group does not take place, the low-energy B theory gauge group is simply connected, and there is no monopole in the B theory.

Furthermore, since the appearance of the $SU(N_f)$ global symmetry in the B theory is quite simple, and is in fact identical to that of the case $N_P = 1$ discussed earlier, we can identify the quantum numbers of the monopole under this symmetry. In particular, if the $Spin(10)$ theory is broken to $Spin(10-k)$ by expectation values for k of the fields V^i, then the $SU(N_f)$ global symmetry is broken to $SU(N_f - k) \times Spin(k)$, with all the spinors P_a transforming as bispinors under $Spin(10-k) \times Spin(k)$. The effect on the B theory is as before; k of the N_f fields Q_i in the antifundamental representation of $SU(N_f + 2N_P - 7)$ develop couplings $Q_i S Q_i$ to the symmetric tensor field S. The flavor symmetry is broken thereby to $SU(N_f - k) \times SO(k)$. When the field S acquires an expectation value and leads to a monopole solution, the k distinguished Q_i develop zero modes in the presence of the monopole, making it a spinor of the global symmetry $SO(k)$, as expected.

The monopole will also transform under the other flavor symmetry of the theory, the one which rotates the N_P spinors into each other. This global symmetry, which is $SU(N_P)$ in the A theory, is reduced to $SU(2)$ in the B theory [9], with the full $SU(N_P)$ only being realized quantum mechanically in the B theory, as a quantum accidental symmetry [13]. The N_P spinors P transform as an N_P dimensional representation (a symmetric combination of $N_P - 1$ doublets) of this $SU(2)$, a remarkable structure not previously seen in duality. Consequently, a prediction of the spinor-monopole identification is that the B model should have zero modes which make the monopole an $(N_P - 1)$-index multispinor under the $SU(2)$ flavor symmetry. It would be a remarkable check on the results of this paper if this pattern of zero modes could be confirmed. To verify it, however, requires an analysis of the complex and intricate breaking pattern in the B model – a challenge for the reader.

3.4. Summary

I have shown in several examples that, where global $Spin(k)$ flavor symmetries arise as a result of symmetry breaking in the A theory, the monopoles of the B theory and the massive spinors of the A theory transform in the same way under them. Furthermore, the monopoles only exist in those theories in which *all* spinors are massive; the presence of even one massless spinor in the A theory eliminates the $SU \to SO$ breaking pattern in the B theory.

4. The physics of various phases

Having established the plausibility of the spinor-monopole identification in these theories, I now turn to a study of their phases. My conclusions will support those of previous authors [14, 1, 2, 10, 3, 4, 15].

I will focus my discussion on the $Spin(8)$ theory and its dual description. (To repeat the analysis for the $Spin(8-k)$ and $Spin(10-k)$ theories considered in the previous section requires minor, inessential modifications.) Throughout this section I will always be referring to the phase of the *low-energy* A theory, that is, the theory of $Spin(8)$ with a certain number of *massless* fields in the vector representation, except when explicitly noted.

4.1. The Coulomb phase

Let us consider, as the A theory, $Spin(8)$ with one spinor P of mass m, M_f vectors V^r of mass m_r, and six massless vectors v^i. This is known as the Coulomb phase of the A theory, since at the generic point in moduli space the six massless vectors break $Spin(8)$ to pure $Spin(2)$ without matter. For non-zero m, m_r, the B theory, with gauge group $SU(M_f+2)$, will be broken to $SO(2)$.

Before studying this theory, it is worth reviewing the results expected when the spinor mass m is infinite but the vector masses m_r are finite [3, 4]. If $m_r = 0$ the B theory is an $SO(M_f+2)$ gauge theory with M_f+6 fields Q_r, q_i in the vector representation. Let us give expectation values to the six fields v^i and masses to the M_f fields V^r. The A theory gauge group becomes $SO(2)$, with M_f massive fields V^r and twenty-seven massive gauge bosons of charge $0, \pm 1$, and some 't Hooft-Polyakov magnetic monopoles. In the B theory, the six fields q_i become massive while the other matter fields develop expectation values $\langle Q^r Q^r \rangle \neq 0$, $r = 1, \ldots, M_f$. This breaks the B theory gauge group to $SO(2)$, with six massive fields q^i and $[(M_f+2)(M_f+1)/2]-1$ massive gauge bosons of charge $0, \pm 1$, and some 't Hooft-Polyakov monopoles. In both descriptions, these monopoles carry magnetic charge ± 1 in units where the minimum Dirac value is $\pm 1/2$. (This magnetic charge is additive, in contrast to the Z_2 monopoles discussed in section 2 and 3 where the charge was only defined mod 2.) From the duality of [4], it can be seen that if the V^r have equal masses and the v^i have equal vacuum expectation values, then the number of monopoles in the A theory is at least six and the number of monopoles of the B theory is at least M_f.

In short, the V^r are visible as monopoles in the B theory while the q_i are visible as monopoles in the A theory. (The extent to which the gauge bosons of the two theories are visible as monopoles has not been fully explored.) The reason that these states are visible, rather than screened, is that there are no massless charged states at the generic point in moduli space. Quantum mechanically, it remains true throughout the entire moduli space that no states electrically charged under the A theory become massless. This is analogous to the physics in $SO(3)$ with a single triplet, also known as pure $\mathcal{N}=2$ $SU(2)$ gauge theory [1]. The only particles which become massless are monopoles and dyons of the A theory. The absence of screening is therefore manifest on the entire Coulomb branch, including at the origin of moduli space where $Spin(8)$

is classically unbroken, and so all electric charges of the A theory become identifiable magnetic charges in the $SO(2)$ B theory.

Let us now consider finite m, with $m \gg m_r$, and repeat this analysis. Since spinors carry electric charges $\pm 1/2$ when the A theory is broken from $Spin(8)$ to $SO(2)$, it is natural to expect that when the B theory is broken from $SU(M_f+2)$ to $SO(2)$, the spinors will appear as monopoles of magnetic charge $\pm 1/2$. To see that the monopoles from the $SU(M_f+2) \to SO(M_f+2)$ breaking have half the charge of those from the $SO(M_f+2) \to SO(2)$ breaking is straightforward. As in the original 't Hooft-Polyakov monopole, both of these monopole solutions involve the winding, in some $SU(2)$ subgroup of the full B theory gauge group, of a field which is a triplet under that $SU(2)$ [16]. In the first case, the triplet is a part of the field S, whose electric charges under the unbroken $SO(2)$ are $\pm 2, 0$. By contrast, the triplet involved in the second monopole is a part of a field Q_i, whose charges are $\pm 1, 0$ under the unbroken $SO(2)$. Thus the two $SU(2)$'s are normalized differently, and it follows that the magnetic charge of the second monopole is twice that of the first. Corroboration is provided by the proof in [7] that two $SU(M_f+2)/SO(M_f+2)$ monopoles either can be completely unwound or can be deformed into an $SO(M_f+2)/SO(2)$ monopole.[4]

In the case $m \gg m_r$ under discussion, where the breaking pattern in the B theory is $SU(M_f+2) \to SO(M_f+2) \to SO(2)$, the monopoles of half-integer charge are much heavier than those of integer charge. This nicely reflects the relative masses of the particles in the A theory with half-integer and integer electric charge. But if $m \ll m_r$, then the spinors are light, the lightest state of charge ± 1 will not be a V^r particle, and we should not see stable heavy monopoles of charge ± 1. This is visible in the B theory, where the expectation values for $Q_i S Q_i$ break the gauge group first to $SU(2)$, generating no monopoles, and then the expectation value for $\det S$ breaks it to $SO(2)$, generating monopoles carrying an additive charge. These monopoles carry magnetic charge $\pm 1/2$, since they are built by winding the field S, and so correspond to spinors. The states of charge ± 1 are light and remain so as $m_r \to \infty$.

Thus, the various limits are consistent with expectations. If we take m to infinity, m_r fixed, we find that the monopoles with half-integer charge become infinitely massive while those of integer charge survive. If we hold m fixed and take m_r to zero, the monopoles with integer charges become light and disappear, while the monopoles with half-integer charge remain as Z_2 monopoles of the non-abelian B theory. If we take m_r to infinity and hold m fixed, states with any magnetic charge survive in the B theory with finite mass. And if we hold m_r fixed and take m to zero, all magnetic monopoles disappear from the B theory, reflecting the expectation that all massive V^r will be screened by the massless P field.

[4] Whether the magnetic charge of a given $SU(N_f-4)/SO(N_f-4)$ monopole under $SO(2)$ is positive or negative depends on its orientation inside the $SO(N_f-4)$ group [7].

4.2. The confining phase

Let us consider the same theory as above, with one modification; let us add a mass $\hat{m}_6$ for v^6. Now, at the generic point in moduli space, there is an unbroken pure $Spin(3)$ gauge group. We expect such a theory to confine throughout the moduli space, and will focus on the point at the origin.

Of course, the test for confinement in $SO(N)$ gauge theories with massless matter in the vector representation is generally to introduce sources charged under the spinor representation of the group. If these sources have a potential energy linear in their separation — or equivalently, if a Wilson loop in the spinor representation has an area law — then the theory is confining. Since the spinor is a magnetic monopole in the B theory, the Wilson loop in the spinor representation of the A theory is directly mapped to a 't Hooft loop in the Z_2 magnetic representation of the B theory. We may now confirm that this theory is confining. In particular, the B theory, which for $\hat{m}_6 = 0$ is an $SU(M_f+2)$ gauge theory broken to $SO(2)$, will be completely broken for non-zero $\hat{m}^6$. As in the Abelian Higgs model, magnetic flux will be confined into Nielsen-Olesen strings [11], resulting in a linear potential for the monopoles. This is consistent with the conventional expectations based on the dual Meissner effect, and agrees with [3, 4]. The massive fields in the vector representation also show confinement at sufficiently short distances.

As before I consider several possible breaking patterns for the B theory, in order to illustrate different aspects of the physics. It is useful to keep in mind the result from the previous section, that both V^r and P are visible in the B theory if $m \gg m_r$, while only P is visible if $m \ll m_r$.

If $m \gg m_r \gg \hat{m}_6$, then both the massive P and V^r particles, previously visible as monopoles in the B theory, should be confined. A string with flux 1/2 should break via pair production of P particles which are very heavy, while an assemblage of strings with integer flux should break at much lower scales. Consequently the confinement of the V^r should break down at distances short compared with the scale at which confinement of P particles is lost.

Indeed this is seen in the B theory, which breaks as $SU(M_f+2) \to SO(M_f+2) \to SO(2) \to 1$. Our previous discussion indicated that there are heavy and light monopoles of half-integral and integral magnetic charge. The breaking of $SO(2)$ by a field Q_6 of integral electric charge confines magnetic flux in a Nielsen-Olesen string configuration, which carries an additive Z charge. The flux carried by the string corresponds to magnetic charge 1/2 under the broken $SO(2)$. The $SU(M_f+2)/SO(M_f+2)$ monopoles are therefore confined by a string of flux 1/2, while those of the $SO(M_f+2)/SO(2)$ theory are confined by either two strings with flux 1/2 or a single string with flux 1, the actual configuration being determined dynamically. When two monopoles of integral charge are pulled far apart, and the total energy becomes very large, the broken $SO(2)$

theory will become sensitive to the fact that it is actually a broken $SO(M_f+2)$ theory, and since $\pi_1[SO(M_f+2)] = Z_2$, the string configuration with total flux 1 will be able to unwind. This represents the screening of the V^r. Only at much higher energies – much longer separations – will the fact that $\pi_1[SU(M_f+2)] = 1$ become important and lead to breaking of the string of flux 1/2. When the spinor mass m is taken arbitrarily large, the $SU(M_f+2) \to SO(M_f+2)$ breaking scale goes to infinity, the Z_2 monopoles become infinitely massive, and the string of flux 1/2 does not break.

What happens if we increase $\hat{m}_6$ until $m \gg m_r \sim \hat{m}_6$? One would expect that the confinement scale increases in energy to the point that the massive V^r particles cannot be far separated before their strings break via pair production, and therefore that the V^r particles and the strings of carrying flux 1 would not be found in the B theory. This expectation is fulfilled. The B theory breaks directly from $SU(M_f+2) \to SO(M_f+2) \to 1$ and its monopoles and Nielsen-Olesen strings carries only a Z_2 charge (since $\pi_1[SO(M_f+2)] = Z_2$); the monopoles and strings associated to the V^r have disappeared. This theory is an explicit and physically interesting realization of the non-abelian generalization of the dual Meissner effect.

By contrast, if $m_r \gg m \gg \hat{m}_6$ then the heavy V^r particles can be screened by pair production of P particles. The $SU(M_f+2) \to SU(2) \to SO(2) \to 1$ breaking pattern in the B theory correspondingly gives only light monopoles bound by Nielsen-Olesen strings with half-integral flux. If we increase $\hat{m}_6$ so that $m_r \gg \hat{m}_6 \gg m$, then we expect the confinement scale in the A theory to be so large that the strings can easily break via pair production of P particles and should not be visible. Indeed, in this limit the B theory breaks directly from $SU(M_f+2)$ to nothing and has neither monopoles nor strings.

In short, duality gives a picture of confinement essentially consistent with conventional expectations, and provides a fully non-abelian example of the dual Meissner effect.

4.3. The free magnetic phase

If the identification of the Z_2 monopole with P is accepted, it can be used to confirm Seiberg's conception of the non-abelian free magnetic phase [14, 10, 3, 15]. If the A theory is $Spin(8)$ with N_f massless fields V^i in the $\mathbf{8_v}$ and a massive spinor P in the $\mathbf{8_s}$, then the B theory is an $SU(N_f-4)$ gauge theory broken to $SO(N_f-4)$, with N_f fields Q_i in the $\mathbf{N_f-4}$ representation and some gauge singlets N^{ij}. For $6 \leq N_f < 9$ the B theory has non-negative beta function, and so is weakly coupled in the infrared.

For $N_f = 6$, the theory is in the Coulomb phase discussed earlier; the B theory has $SO(2)$ gauge symmetry. Far out along the moduli space, where the A theory is broken to $SO(2)$ also, ordinary Maxwell electric-magnetic duality implies that electrically charged sources carrying spinor charge in the A theory will appear as magnetically charged sources in the B theory of charge $\pm 1/2$ [2, 3, 4]. One may then argue that this identification can be carried to the origin of moduli space, where Maxwell duality cannot be directly used. The spinor-monopole identification lends further credence to this argument, since as shown earlier the spinor indeed appears as a monopole of charge $\pm 1/2$. It is known from $SO(N)$ duality that at the origin of moduli space, the six fields Q_i carrying electric charge under the B theory become light, causing the gauge coupling of the B theory to run logarithmically to zero at long distance. Consequently, the coupling governing interactions of magnetic sources goes logarithmically to infinity. We may conclude that the interaction between two static spinor particles separated by a large distance r takes the form $\log r/r$, as argued in [15].[5]

For $N_f > 6$ the situation has previously been less clear. Far from the origin of moduli space, the A theory is broken to $SO(2)$ with massless matter. Because of the light charged fields, classical Maxwell electric-magnetic duality cannot be used. Interpolation to the origin of moduli space is therefore nontrivial. However, the arguments of this paper resolve any outstanding issues. The B theory has non-negative beta function, so at the origin of moduli space its gauge coupling flows logarithmically to zero at long distance. The interaction energy between two static spinor sources in the A theory is just that of two semiclassical Z_2 monopoles of the B theory; it behaves as $\log r/r$ with a computable coefficient.

A comment about the difference between the confining phase and the free magnetic phase should be made. The presence of the massless mesons N^{ij}, which are weakly coupled massless particles of the B theory in the free magnetic, Coulomb and confining phases, might lead at first glance to the misconception that electric charges in the A theory are confined. It is tempting to think of N^{ij}, which carries the flavor quantum numbers of a bound state of two V^i particles, as a confined system. However, this image is clearly inaccurate, as we have just seen. The presence of the massless mesons is instead closely related to anomalies and chiral symmetries. Similar analysis applies to the real-world pion system; it is well understood that the presence of light pions does not in any way imply confinement.

Perhaps this is also a good place to address the question of whether it is reasonable in $\mathcal{N} = 1$ supersymmetry to trust the identification of the massive monopole with the massive spinor. Ideally, there would be a limit in which the B theory was weakly coupled at the scale of the monopole mass, so that one could show that the B theory description of the A theory really did contain a monopole solution. However, such a limit does not exist. As I will now show, this follows from the fact that the $Spin(8)$ theory with a massless spinor does not have a free magnetic phase; *i.e.*, that the B

[5]Note that along the part of the moduli space where $Spin(8)$ is broken to $Spin(3)$, the theory is related to the $\mathcal{N} = 2$ theories studied by Seiberg and Witten [1, 2]; the discussion of this paragraph is of course consistent with their results.

theory with unbroken $SU(N_f-4)$ gauge group always has a negative beta function.[6] After explaining the problem, I will give the strongest argument that I can construct.

First, consider the following physical situation: at high energies the A theory description is weakly coupled, and there is a spinor with mass m large compared with the scale Λ_A where the A theory coupling becomes strong. The question is whether the B theory has a monopole with the same properties as the A theory spinor. Of course, when we convert to B theory variables, they are strongly coupled at the scale of order m where $SU(N_f-4)$ breaks to $SO(N_f-4)$, as they must be, since A theory perturbation theory works there. Only after the breaking takes place does the B theory begin to flow toward weak coupling. Thus, for large m, we cannot trust B theory semiclassical arguments about soliton solutions. It is then natural to ask if by lowering the mass m we can reach a regime where semiclassical reasoning in the B theory will work and give a magnetic monopole via the $SU(N_f-4) \to SO(N_f-4)$ breaking. If this were so, then we would expect that as m is taken larger, the details of the monopole solution would no longer be given by semiclassical B theory physics, but the existence of at least one heavy, magnetically charged state would be ensured. Unfortunately, the B theory does not become weakly coupled when m is lowered, since the beta function of the $SU(N_f-4)$ group is negative. As a result, for small m, semiclassical physics fails completely; the spinor is too light to be treated perturbatively in the A theory, while the B theory remains strongly coupled at the $SU(N_f-4) \to SO(N_f-4)$ breaking mass scale, making a semiclassical soliton analysis impossible. In fact, as $m \to 0$ the theory flows to a non-trivial conformal fixed point.

Nevertheless, an argument from a different point of view can still be given, using the fact that the *low-energy* B theory is weakly coupled. Although semiclassical reasoning at short distances is unreliable, topological reasoning at long distances is trustworthy. Topology implies that the low-energy $SO(N_f-4)$ gauge theory can in principle have a Z_2 monopole, since $\pi_1[SO(N_f-4)] = Z_2$ implies the consistency of a Z_2 Dirac string. Independently of duality, semiclassical arguments show that if the B theory is weakly coupled at the scale where $SU(N_f-4)$ breaks to $SO(N_f-4)$, then a soliton with long-range magnetic fields, whose total flux is conserved, certainly exists. If we then consider increasing the coupling of the $SU(N_f-4)$ theory to a large value, as needed to match onto the physics of the A theory, the details of the monopole, such as its mass and core shape, will change in an uncontrollable way; it may even decay to lighter states. But despite these changes there must still exist some state in the theory charged under a Z_2 and surrounded by a long-range $SO(N_f-4)$ magnetic field. This state must be both heavy and small, since otherwise it will be in conflict with the weakly coupled physics of $SO(N_f-4)$. For the arguments used in this study of phases, merely the existence of such an object is needed.

[6] The A theory has a free electric phase, and so the reverse argument *does* work, as will be shown in the next section.

4.4. The free electric phase and large N_f

Again consider $Spin(8)$ with N_f massless fields V^i in the $\mathbf{8_v}$ and a massive spinor P in the $\mathbf{8_s}$. If $N_f \geq 18$, the $Spin(8)$ theory loses asymptotic freedom. The interesting physical situation is then given by taking the B theory as weakly coupled in the far ultraviolet; below its strong coupling scale Λ_B it flows to the weakly coupled A theory. To make the B theory renormalizable, we should consider it with superpotential $W = 0$; the resulting adjustment to the duality, given in the Appendix, changes none of the physics relevant for this discussion.

The behavior of the 't Hooft loop of the B theory is familiar. The potential energy at long distance between two ultraheavy spinors/monopoles is given by the weak coupling physics of the A theory as $1/(r \log r)$, as for electrons in massless QED.

I will now argue that the spinor/monopole of finite mass is well described as a monopole of the B theory when it is heavy, and well described as a weakly coupled spinor of $Spin(8)$ when it is light.

Let us go along a flat direction $\langle \det S \rangle = v_0^{N_f - 4}$ such that the B theory breaks to $SO(N_f - 4)$ at the scale v_0. If $v_0 \gg \Lambda_B$, then this breaking occurs at weak coupling, and the semiclassical monopole solution can be trusted. In the A theory, the mass of this state is much larger than Λ_A (the scale of the perturbative Landau pole of the A theory) and so the details of its structure are lost in a strongly coupled fog. However, its long-range fields extend into the weakly coupled regime of the A theory, and as they are unscreened, they must be the electric fields of a massive spinor of $Spin(8)$. Thus, from the point of view of the low-energy $Spin(8)$ theory, the monopole of the B theory will act as a ultramassive particle with quantum numbers of an $\mathbf{8_s}$ representation.

As the scale v_0 is taken smaller and the monopole becomes lighter, the B theory becomes strongly coupled and the semiclassical description of the monopole will gradually worsen. Meanwhile the massive $\mathbf{8_s}$ particle of the A theory cannot decay, since it carries a conserved Z_2 quantum number, and it must survive as a light particle of the weakly coupled low-energy A theory. General renormalization group considerations ensure that, if it is light enough, its properties will be those of an ordinary particle — for example, its kinetic terms will be canonical. Where duality makes its strongest statement is that this particle becomes massless as v_0 goes to zero. Anomalies and other symmetry considerations make this possible, and in a non-trivial way, necessary.

We therefore see that for large N_f the heavy monopole description gradually and smoothly goes over to the light spinor description as v_0 is taken from large to small. This uneventful transition between two controllable regimes is possible because the B theory has a free magnetic phase even when the spinor is massless, and so reliable weakly coupled descriptions for the theory and its spinor/monopole exist both in the ultraviolet and in the infrared.

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4.5. The non-Abelian Coulomb phase

Finally, consider $Spin(8)$ with $7 \leq N_f \leq 17$ massless fields V^i in the $\mathbf{8_v}$ and a massive spinor P in the $\mathbf{8_s}$. In this case, the low-energy theory is a non-trivial conformal field theory. The potential energy between spinor/monopole sources is $1/r$, which follows from conformal invariance.

No direct construction of these conformal field theories has been found, and many of their properties have not been characterized. As a result, it is impossible to say much about the effect of the massive spinor/monopole on the low-energy theory, other than to note its presence will generate various irrelevant perturbations, whose form is constrained by symmetries, on the low-energy fixed point. Perhaps in the future we will learn how to make more useful statements about this physical situation.

5. Final remarks

It would be enlightening to have many more examples of similar phenomena. An example of a theory which would be interesting to understand is $Spin(8)$ with N_f fields in the $\mathbf{8_s}$ and one in the $\mathbf{8_c}$ or $\mathbf{8_v}$, constructed by perturbing the $Spin(10)$ theory with multiple fields in the **16** and **10**. Although the A theory is the same, up to $Spin(8)$ triality, as the $Spin(8)$ theory studied in earlier sections, the B theory description will be very different. It would be interesting to see if the $\mathbf{8_v}$ or $\mathbf{8_c}$ are visible as monopoles in the dual theory, and if so, through what symmetry breaking pattern. Unfortunately, the B theory for $Spin(10)$ with multiple spinors is complicated by the presence of quantum accidental symmetries [3, 13, 9], and this construction has not yet been performed.

However, there is in general no *a priori* reason to expect any given $\mathcal{N} = 1$ supersymmetric duality transformation to map a massive field to a topologically stable soliton. The criteria under which a particle of one theory will appear in the semiclassical physics of the other have not yet been understood. The absence of BPS charges for particles means that there are usually no rigorous arguments. It may be hoped that future work in string theory will improve our understanding of these issues.

The implications of the spinor-monopole identification for string theory and M theory constructions of $\mathcal{N} = 1$ duality are worthy of note. The first chiral gauge theories from Type IIA string theory and M theory branes are being constructed at the time of this writing [17]. The fact that spinors in $Spin(N)$ appear as non-BPS monopoles of chiral gauge theories may be a useful hint for the Type IIA or M theory construction of this duality. Furthermore, it may portend a substantial number of results to come involving non-BPS solitons in field theory, string theory and M theory, along the lines of the non-BPS strings found in the M theory construction of QCD [18]. At the very least, it will be interesting to understand how the relations found in this paper are manifested in the Type IIA/M theory construction.

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Finally, many $\mathcal{N}=1$ gauge theories have yet to be fully understood. The question of whether there are other unusual or perhaps misidentified phases of these theories remains open. The ability to map Wilson loops into the dual description of a theory is critical for testing its properties. It may be hoped that further work in the directions suggested by this paper will lead to greater insight into the phase structure of gauge theories, and into the nature of duality itself.

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A. Duality transformations for $Spin(N)$

The five duality transformations used in this paper are summarized. Only essential elements are presented; the reader is directed to the original references for more details.

A.1. $Spin(N)$ with N_f vectors

The A theory has gauge group $SO(N)$; the B theory has gauge group $SO(N_f-N+4)$ [3]. They share an $SU(N_f)$ global symmetry. The matter content of the theory is as follows:

A theory	$SO(N)$	$[SU(N_f)]$	B theory	$SO(N_f-N+4)$	$[SU(N_f)]$
V^i	$\mathbf{N}$	$\square$	Q_i	$\square$	$\bar{\square}$
			N^{ij}	$\mathbf{1}$	$\square\square$

Under duality the following chiral operators are identified:

V^iV^j	$\leftrightarrow$	N^{ij}
$(V)^{N-2k}W_\alpha^k$	$\leftrightarrow$	$(Q)^{N_f-N+2k}\tilde{W}_\alpha^{2-k}$

The superpotential of the A theory is zero, while that of the B theory, setting its coefficient to one, is

$$W = N^{ij}Q_iQ_j\,. \tag{A.1}$$

A.2. $Spin(8)$ with N_f vectors and one spinor

The A theory has gauge group $Spin(8)$; the B theory has gauge group $SU(N_f - 4)$ [6]. They share an $SU(N_f)$ global symmetry. The matter content of the theory is as follows:

A theory	$Spin(8)$	$[SU(N_f)]$	B theory	$SU(N_f-4)$	$[SU(N_f)]$
V^i	$\mathbf{8_v}$	$\square$	S	$\square\!\square$	$\mathbf{1}$
P	$\mathbf{8_c}$	$\mathbf{1}$	Q_i	$\overline{\square}$	$\overline{\square}$
			N^{ij}	$\mathbf{1}$	$\square\!\square$
			T	$\mathbf{1}$	$\mathbf{1}$

Under duality the following chiral operators are identified:

V^iV^j	$\leftrightarrow$	N^{ij}
PP	$\leftrightarrow$	T
$(V)^4P^2$	$\leftrightarrow$	$(Q)^{N_f-4}$

The superpotential of the A theory is zero, while that of the B theory, setting all coefficients to one, is

$$W = N^{ij}Q_iSQ_j + T\det S\,. \tag{A.2}$$

A.3. $Spin(8)$ with N_f vectors and one spinor along with gauge singlets [6]

This theory is a trivial modification of the previous one [6]. The matter content of the theory is as follows:

A theory	$Spin(8)$	$[SU(N_f)]$	B theory	$SU(N_f-4)$	$[SU(N_f)]$
V^i	$\mathbf{8_v}$	$\square$	S	$\square\!\square$	$\mathbf{1}$
P	$\mathbf{8_c}$	$\mathbf{1}$	Q_i	$\overline{\square}$	$\overline{\square}$
M_{ij}	$\mathbf{1}$	$\overline{\square\!\square}$			
U	$\mathbf{1}$	$\mathbf{1}$			

Under duality the following chiral operators are identified:

M_{ij}	$\leftrightarrow$	Q_iSQ_j
U	$\leftrightarrow$	$\det S$
$(V)^4P^2$	$\leftrightarrow$	$(Q)^{N_f-4}$

The superpotential of the B theory is zero, while that of the A theory, setting all coefficients to one, is

$$W = M_{ij}V^iV^j + UPP\,. \tag{A.3}$$

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A.4. $Spin(10)$ with N_f vectors and one spinor

The A theory has gauge group $Spin(10)$; the B theory has gauge group $SU(N_f-5)$ [8]. They share an $SU(N_f)$ global symmetry. The matter content of the theory is as follows:

A theory	$Spin(10)$	$[SU(N_f)]$	B theory	$SU(N_f-5)$	$[SU(N_f)]$
V^i	$\mathbf{10}$	$\square$	S	$\square\!\square$	$\mathbf{1}$
P	$\mathbf{16}$	$\mathbf{1}$	Q_i	$\overline{\square}$	$\overline{\square}$
			F	$\square$	$\mathbf{1}$
			N^{ij}	$\mathbf{1}$	$\square\!\square$
			Y^i	$\mathbf{1}$	$\square$

Under duality the following chiral operators are identified:

V^iV^j	$\leftrightarrow$	N^{ij}
V^iPP	$\leftrightarrow$	Y^i
$(V)^5(P)^2$	$\leftrightarrow$	$(Q)^{N_f-5}$

The superpotential of the A theory is zero, while that of the B theory, setting all coefficients to one, is

$$W = \det S + N^{ij}Q_iSQ_j + Y^iQ_iF\,. \tag{A.4}$$

A.5. $Spin(10)$ with N_f vectors and $N_P > 1$ spinors

The A theory has gauge group $Spin(10)$; the B theory has gauge group $SU(\tilde{N}) \times Sp(2\tilde{M})$, where $\tilde{N} = N_f + 2N_P - 7$ and $\tilde{M} = (N_P - 1)$ [9]. They share an $SU(N_f)$ global symmetry. The $SU(N_P)$ symmetry which rotates the spinors is a quantum accidental symmetry [13] in the B theory, which classically has only an $SU(2)$ subgroup of this symmetry. This $SU(2)$ is embedded in $SU(N_P)$ such that the $\mathbf{N_P}$ representation of the former is the $\mathbf{N_P}$ representation of the latter. For simplicity I indicate the transformation properties of operators only under the $SU(2)$ subgroup of $SU(N_P)$.

The matter content of the theory is as follows:

A theory	$Spin(10)$	$[SU(N_f)]$	$[SU(2)]$	B theory	$SU(\tilde{N})$	$Sp(2\tilde{M})$	$[SU(N_f)]$	$[SU(2)]$
V^i	$\mathbf{10}$	$\square$	$\mathbf{1}$	S	$\square\!\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
P_I	$\mathbf{16}$	$\mathbf{1}$	N_P	Q_i	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
				F^r	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2N_P-1}$
				Q'_a	$\overline{\square}$	$\square$	$\mathbf{1}$	$\mathbf{2}$
				t_X	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{2N_P-2}$
				N^{ij}	$\mathbf{1}$	$\mathbf{1}$	$\square\!\square$	$\mathbf{1}$
				Y^i_r	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{2N_P-1}$

Under duality the following chiral operators are identified:

V^iV^j	$\leftrightarrow$	N^{ij}
$V^i(P)^2$	$\leftrightarrow$	$Y^i_a, (Q)^{N_f-1}(Q')^{2N_P-6}$
$(V)^3(P^2)$	$\leftrightarrow$	$(Q)^{N_f-3}(Q')^{2N_P-4}$
$(V)^5(P)^2$	$\leftrightarrow$	$(Q)^{N_f-5}(Q')^{2N_P-2}$
$(P)^4$	$\leftrightarrow$	$(t)^2, \ldots$

The superpotential of the A theory is zero, while that of the B theory, setting all coefficients to one, is

$$W = N^{ij}Q_iSQ_j + Y^i_rQ_iF^r + Q'_aSQ'_b\epsilon^{ab} + Q'_at_XF^r\,. \tag{A.5}$$

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